

# Supersymmetric Models with Approximate CP

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WIS-99/27/OCT-DPP

Supersymmetric models with an approximate CP,  $10^{-3} \lesssim \phi_{CP} \ll 1$ , are a viable framework for the description of nature. The full high energy theory has exact CP and horizontal symmetries that are spontaneously broken with a naturally induced hierarchy of scales,  $\Lambda_{CP} \ll \Lambda_H$ . Consequently, the effective low energy theory, that is the supersymmetric Standard Model, has CP broken explicitly but by a small parameter. The  $\varepsilon_K$  parameter is accounted for by supersymmetric contributions. The predictions for other CP violating observables are very different from the Standard Model. In particular, CP violating effects in neutral B decays into final CP eigenstates such as  $B \rightarrow \psi K_S$  and in  $K \rightarrow \pi \nu \bar{\nu}$  decays are very small. This framework, though, is strongly disfavored by the recent measurements of  $\varepsilon'_K/\varepsilon_K$ .

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# 1 Introduction and Motivation

Only two CP violating parameters have been measured to high accuracy so far [1]-[5]:

$$\varepsilon_K = (2.280 \pm 0.013) \times 10^{-3} e^{i\frac{\pi}{4}} \quad (1)$$

$$Re(\varepsilon'_K/\varepsilon_K) = (2.11 \pm 0.46) \times 10^{-3}, \quad (2)$$

where

$$\varepsilon_K = \frac{\langle (\pi\pi)_{I=0} | \mathcal{L}_W | K_L \rangle}{\langle (\pi\pi)_{I=0} | \mathcal{L}_W | K_S \rangle}, \quad (3)$$

$$Re(\varepsilon'_K/\varepsilon_K) = \frac{1}{6} \left( \left| \frac{\langle \pi^+\pi^- | \mathcal{L}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{L}_W | K_S \rangle} \frac{\langle \pi^0\pi^0 | \mathcal{L}_W | K_S \rangle}{\langle \pi^0\pi^0 | \mathcal{L}_W | K_L \rangle} \right|^2 - 1 \right). \quad (4)$$

Within the Standard Model (SM) the value of  $\varepsilon_K$  can be accounted for if the single CP violating phase,  $\delta_{KM}$  in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, is of  $O(1)$ . Although the phase is large, the effect is small due to flavor parameters.

The theoretical interpretation of  $\varepsilon'_K/\varepsilon_K$  suffers from large hadronic uncertainties. The SM theoretically preferred range is somewhat lower than the experimental range (for recent work, see refs. [6, 7] and references therein). Yet, if all the hadronic parameters are taking values at the extreme of their reasonable ranges, the experimental result can be accommodated.

There are CP violating observables that have not yet been accurately measured, for example:

$$a_{\psi k_s} \sin(\Delta m_B t) = \frac{\Gamma(\bar{B}_{phys}^0(t) \rightarrow \psi K_S) - \Gamma(B_{phys}^0(t) \rightarrow \psi K_S)}{\Gamma(\bar{B}_{phys}^0(t) \rightarrow \psi K_S) + \Gamma(B_{phys}^0(t) \rightarrow \psi K_S)}, \quad (5)$$

$$a_{\pi\nu\bar{\nu}} = \frac{\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})}. \quad (6)$$

The values of these observables are predicted within the SM to be [8, 9]:

$$(a_{\psi k_s})_{SM} = 0.4 - 0.8, \quad (7)$$

$$(a_{\pi\nu\bar{\nu}})_{SM} = O(0.2). \quad (8)$$

The smallness of the measured parameters (1)-(2), however, suggests that there might be new physics that allows a viable description of CP violating phenomena with approximate CP, that is with all CP violating phases smaller than  $O(1)$ . In such a framework it is possible that the predictions for other CP violating observables are substantially different from the SM. In particular,  $a_{\psi k_s}$  and  $a_{\pi\nu\bar{\nu}}$  are both much smaller than one.

Below we present a framework where the idea of approximate CP is realized. This framework was introduced in ref. [10], where two explicit supersymmetric (SUSY) models were given. Here, all the CP violating phases are small. In particular  $\delta_{KM}$  is small, and  $\varepsilon_K$  is accounted for by new physics, requiring at least one phase larger than or of  $O(10^{-3})$ . We also report the results of a recent reexamination of this framework [11], in light of the accurate measurement of  $\varepsilon'_K/\varepsilon_K$ .

## 2 The Framework

Our high-energy theory is supersymmetric and has CP and abelian horizontal symmetries [12]. At low energies we assume that SUSY is softly broken. Generic values for SUSY parameters might lead to too large flavor changing neutral currents (FCNC). In our framework we use the breaking of the horizontal symmetry supplemented by the mechanism of alignment [13] to avoid this problem. In order to account for CP violation, we break CP spontaneously in such a way that in the low energy effective theory the CP violating phases are small. With approximate CP, the potential CP problem of SUSY models, that is too large contributions to electric dipole moments (EDM) [14], is avoided.

Below we describe in more detail the various ingredients of our framework.

### 2.1 Abelian Horizontal Symmetry

Models of abelian horizontal symmetries are able to provide a natural explanation for the hierarchy in the quark and lepton flavor parameters [12, 15]. The full high energy theory has an exact horizontal symmetry,  $H$ . The superfields of the supersymmetric standard model (SSM) carry  $H$ -charges. In addition there is usually at least one SM singlet superfield,  $S$ , that also carries  $H$ -charge. The horizontal symmetry is spontaneously broken when the SM singlet field assumes a vacuum expectation value (vev),  $\langle S \rangle$ . The breaking scale is somewhat lower than a scale  $M$  where the information about this breaking is communicated to the SSM, presumably by heavy quarks in vector like representations of the SM (the Froggatt-Nielsen mechanism [12]). The smallness of the ratio between the two scales,  $\lambda \sim \frac{\langle S \rangle}{M} \ll 1$ , is the source of smallness and hierarchy in the Yukawa couplings. The parameter  $\lambda$  is taken to be of order of the Cabibbo angle,  $O(0.2)$ . Models are defined by the horizontal symmetry, the assigned horizontal charges and the hierarchy of vevs. For most purposes it is sufficient to analyze the effective low energy theory, which is the SSM supplemented with the following selection rules:

- (i) Terms in the superpotential that carry charge  $n$  under  $H$  are suppressed by  $\lambda^n$  if  $n \geq 0$  and vanish otherwise.
- (ii) Terms in the Kähler potential that carry charge  $n$  under  $H$  are suppressed by  $\lambda^{|n|}$ .

These selection rules allow estimation of the various entries in the quark mass matrices  $M^q$  and the squark mass-squared matrices  $M_q^2$  (the coefficients of  $O(1)$  which appear in each entry are not known). The size of the bilinear  $\mu$  and  $B$  terms can also be estimated. From the mass matrices, one can further estimate the mixing parameters in the CKM matrix and in the gaugino couplings to quarks and squarks.

A convenient way to parameterize SUSY contributions to various processes is by using the  $(\delta_{MN}^q)_{ij}$  parameters. In the basis where quark masses and gluino couplings are diagonal, the dimensionless  $(\delta_{MN}^q)_{ij}$  parameters stand for the ratio between  $(M_q^2)_{ij}^{MN}$ , the  $(ij)$  entry ( $i, j = 1, 2, 3$ ) in the squark mass-squared matrix ( $M, N = L, R$  and  $q = u, d$ ), and  $\tilde{m}^2$ , the average squark mass-squared. If there is no mass degeneracy among squarks, then these parameters can be related to the SUSY mixing angles.

The naive values of the different parameters can be calculated using the horizontal

symmetry  $U(1)$ . For example, the naive estimate of the  $(\delta_{LR}^d)_{12}$  parameter which is relevant to  $\varepsilon'_K/\varepsilon_K$  is given by:

$$(\delta_{LR}^d)_{12} \sim \frac{(M_d^2)_{12}^{LR}}{\tilde{m}^2} \sim \frac{\tilde{m} M_{12}^d}{\tilde{m}^2} \sim \frac{m_s |V_{us}|}{\tilde{m}} \sim \lambda^6 \frac{m_t}{\tilde{m}}. \quad (9)$$

## 2.2 Alignment

The naive suppression of the supersymmetric flavor changing couplings is not strong enough to solve all the SUSY FCNC problems. To solve the  $\Delta m_K$  problem, one can use the horizontal symmetry and holomorphy to induce a very precise *alignment* of the quark mass matrices and the squark mass-squared matrices [13, 16], resulting in a very strong suppression of the relevant mixing angles in the gaugino couplings to quarks and squarks. In order to achieve alignment, some of the entries in  $M^d$  should be suppressed compared to their naive values. The required suppression is achieved by the use of holomorphy that causes some of the Yukawa couplings to vanish [16]. In order to achieve this, more than one  $U(1)$  horizontal symmetry is required. These holomorphic zeroes are lifted when the kinetic terms are canonically normalized [16], but their values are suppressed by at least a factor of  $\lambda^2$  relative to their naive value [10].

Returning to our example we now find (horizontal symmetry + alignment):

$$(\delta_{LR}^d)_{12} \sim \frac{\tilde{m} M_{12}^d}{\tilde{m}^2} \lesssim \lambda^2 \frac{m_s |V_{us}|}{\tilde{m}} \sim \lambda^8 \frac{m_t}{\tilde{m}}. \quad (10)$$

## 2.3 Spontaneous CP Breaking

As stated above, the high energy theory is CP symmetric. CP is spontaneously broken in the following way. There are two SM singlet superfields,  $S_1$  and  $S_2$ , that carry charges under the same  $U(1)$  horizontal symmetry. Both of them receive vevs,  $\langle S_2 \rangle \ll \langle S_1 \rangle$ . While one of the vevs can be chosen to be real, the second is in general complex, with a phase of  $O(1)$ . The hierarchy between the vevs and the relative,  $O(1)$  phase, are naturally induced in this framework [17]. This complex vev feeds down to all the couplings.

In the low energy effective theory, there are many independent CP violating phases, in particular in the mixing matrices of gaugino couplings to fermions and sfermions. Furthermore, the ratio of vevs enables all CP violating phases to be suppressed, giving approximate CP. The suppression of phases in the effective theory is by even powers of the breaking parameter.

Returning to our example we find in this case (horizontal symmetry + alignment + approximate CP):

$$Im(\delta_{LR}^d)_{12} \lesssim \lambda^4 \frac{m_s |V_{us}|}{\tilde{m}} \sim \lambda^{10} \frac{m_t}{\tilde{m}}. \quad (11)$$

### 3 Models and Predictions

In ref. [10] two representative models of approximate CP were constructed. One of the models (model II) has the smallest viable CP breaking parameter of  $O(0.001)$ , and the other (model I) has an intermediate value of  $O(0.04)$ .

Regarding FCNC processes, we find in our models:

(i) The contributions to  $\Delta m_D$  saturate the experimental upper bound in both models. This is a generic feature of models of alignment, related to the fact that in these models the Cabibbo mixing ( $|V_{us}| \sim \lambda$ ) comes from the up sector.

(ii) The contributions to  $\Delta m_B$  are very small.

(iii) The contributions to  $\Delta m_K$  are of  $O(10\%)$  in model I and saturate the experimental value for model II. This is in contrast to all previous models of alignment where, to satisfy the  $\varepsilon_K$  constraint, SUSY contributions to  $\Delta m_K$  were negligibly small.

(iv) The contributions to other FCNC processes, such as  $\Delta m_{B_s}$  and  $b \rightarrow s\gamma$ , are very small. As concerns the rare  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay, in both our models the SUSY contributions are of  $O(10\%)$ . While both the SM and the SUSY amplitudes are real to a good approximation, so that there is maximal interference between the two, the relative sign is unknown so that the rate could be either enhanced or suppressed compared to the SM.

In both models  $\varepsilon_K$  is accounted for by SUSY gluino-mediated diagrams [18]. Our results concerning CP violation are summarized in table 1 where  $a_{\psi K_S}$  and  $a_{\pi \nu \bar{\nu}}$  are defined above, and  $d_N$  is the EDM of the neutron (given in units of  $10^{-23} e \text{ cm}$ , so that the present experimental bound is  $d_N \lesssim \lambda^2$ ).

Process	SM	Model I	Model II
$a_{\psi K_S}$	$O(1)$	$O(\lambda^2) \sim 0.04$	$O(\lambda^4) \sim 10^{-3}$
$a_{\pi \nu \bar{\nu}}$	$O(\lambda) \sim 0.2$	$O(\lambda^4) \sim 10^{-3}$	$O(\lambda^8) \sim 10^{-6}$
$d_N$	0	$O(\lambda^4) \sim 10^{-3}$	$O(\lambda^6) \sim 6 \times 10^{-5}$

Table 1: CP violating observables in the SM and in our models.

### 4 $\varepsilon'_K/\varepsilon_K$

With approximate CP  $\delta_{KM}$  is small and SM contributions can not account for the experimental measurement of  $\varepsilon'_K/\varepsilon_K$ . New physics is required. (If the relevant hadronic matrix element is much larger than its value in the vacuum insertion approximation, as suggested by a recent lattice calculation [19], then the SM contribution with a small value of  $\delta_{KM}$  can account for  $\varepsilon'_K/\varepsilon_K$  [20].)

For SUSY to account for  $\varepsilon'_K/\varepsilon_K$ , at least one of the following conditions should be satisfied [18],[21]-[24]:

$$Im[(\delta_{LL}^d)_{12}] \sim \lambda \left( \frac{\tilde{m}}{500 \text{ GeV}} \right)^2,$$

$$Im[(\delta_{LR}^d)_{12}] \sim \lambda^7 \left( \frac{\tilde{m}}{500 \text{ GeV}} \right), \quad (12)$$

$$\begin{aligned} Im[(\delta_{LR}^d)_{21}] &\sim \lambda^7 \left( \frac{\tilde{m}}{500 \text{ GeV}} \right), \\ Im[(\delta_{LR}^u)_{13}(\delta_{LR}^u)_{23}^*] &\sim \lambda^2, \\ Im[V_{td}(\delta_{LR}^u)_{23}^*] &\sim \lambda^3 \left( \frac{M_2}{m_W} \right), \\ Im[V_{ts}^*(\delta_{LR}^u)_{13}] &\sim \lambda^3 \left( \frac{M_2}{m_W} \right). \end{aligned} \quad (13)$$

In our framework only the conditions involving  $(\delta_{LR}^d)_{12}$  and  $(\delta_{LR}^d)_{21}$  can be met. Checking what are the lower bounds on these parameters for extreme values of the parameters (for details see ref. [11]) we find:

$$Im(\delta_{LR}^d)_{12} \gtrsim 7 \times 10^{-7}, \quad (14)$$

that is  $O(\lambda^9)$  or even  $O(\lambda^{10})$  if  $\lambda \sim 0.24$ . A similar bound applies to  $Im[(\delta_{LR}^d)_{21}]$ .

In models in which the flavor problems are solved by alignment, but the CP problems are solved by approximate CP, eq. (11) holds. This is consistent with the experimental constraint of eq. (14) only if all the following conditions are simultaneously satisfied:

- (i) The suppression of the relevant CP violating phases is ‘minimal’, that is  $O(\lambda^2)$ .
- (ii) The alignment of the first two down squark generations is ‘minimal’, that is  $O(\lambda^2)$ .
- (iii) The mass scale of the SUSY particles is low,  $\tilde{m} \sim 150 \text{ GeV}$ .
- (iv) The hadronic matrix element is larger than what hadronic models suggest.
- (v) The mass of the strange quark is at the lower side of the theoretically preferred range.
- (vi) The value of  $\varepsilon'_K/\varepsilon_K$  is at the lower side of the experimentally allowed range.

We conclude that models that combine alignment and approximate CP are disfavored by the measurement of  $\varepsilon'_K/\varepsilon_K$ . More than that, the explicit models (model I and II) described above are ruled out by this measurement.

We do note, however, that models of abelian horizontal symmetries and approximate CP where the flavor problems are solved by a mechanism different from alignment can account for  $\varepsilon'_K/\varepsilon_K$ .

## 5 Conclusions

In the near future, we expect first measurements of various CP asymmetries in  $B$  decays, such as  $B \rightarrow \psi K_S$  or  $B^\pm \rightarrow \pi^0 K^\pm$ . If these asymmetries are measured to be of order one, it will support the SM picture, that the CP violation that has been measured in the neutral  $K$  decays is small because it is screened by small mixing angles, while the idea that CP violation is small because all CP violating phases are small will be excluded. It is interesting, however, that various specific models that realize the latter idea, such as those discussed in this work, can already be excluded by the measurement of a tiny CP violating effect,  $\varepsilon'_K \sim 5 \times 10^{-6}$ .

## Acknowledgments

I thank Antonio Masiero, Yossi Nir and Luca Silvestrini for enjoyable collaborations on the topics presented here.

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